# A Fast and Scalable Algorithm for Calculating the Achievable Capacity of a Wireless Mesh Network

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***Abstract*—This paper considers the problem of rapidly determining the maximum achievable capacity of a multi-hop wireless mesh network subject to interference constraints. Being able to quickly determine the maximum supported flow in a wireless network has numerous practical applications for network planners and researchers, including quickly visualizing capacity limits of a wireless network, benchmarking the performance of wireless protocols, and rapidly determining the instantaneous capacity of various network topologies or different snapshots of a mobile network. Current approaches for determining network capacity either provide asymptotic results that are not necessarily achievable, are computationally intractable and cannot be computed quickly, or are not generalizable to different interference constraints for emerging technologies. In this paper, we present a new algorithm to rapidly determine the maximum concurrent flow for an arbitrary number of unicast and multicast connections subject to general interference constraints, and provide a feasible route and schedule for those flows. The solution provided by our algorithm is within $O(\delta)$ of the optimal maximum flow, where $\delta$ is the maximum number of users that are blocked from receiving due to interference from a given transmission. We then use our algorithm to perform a network capacity analysis comparing different wireless technologies, including omni-directional, directional, and multi-beam directional networks.**

## I. Introduction

As the number of wireless devices experiences explosive growth, there is continued effort in developing protocols to interconnect these devices using multi-hop networks. A key metric in assessing the performance of these protocols is to understand their achieved network capacity with respect to the maximum capacity that a particular network can support. While significant work has gone into understanding wireless network scalability [1], developing cross-layer optimization schemes for achieving near-optimal network throughputs [2], or determining bounds on network capacity [3], there still does not exist an approach that can rapidly determine the achievable capacity of a wireless network under an arbitrary set of interference constraints within a fixed bound of the optimal solution. Such a tool would have numerous applications for a network planner or researcher, enabling the determination of the instantaneous capacity with respect to time of a wireless network with known mobility patterns, quickly assessing the achievable maximum flow of a large number of potential network topologies or deployments, comparing achievable flows for different wireless technologies and their respective interference patterns against one another, and understanding how well wireless protocols perform versus the achievable capacity.

In this paper, we present a new algorithm that can rapidly determine the maximum achievable concurrent flow for any number of unicast and multicast connections within a wireless network subject to general interference constraints. In particular, we consider networks with known parameters, such as node placements, transmission distances, link capacities, and interference constraints. The algorithm finds a set of interference-free routes and transmission schedules for all of the connections in the network such that the minimum rate of any particular connection is maximized. In general, finding an optimal transmission schedule is NP-Hard [4]. Our algorithm runs in polynomial time and achieves a network capacity that is $O(\delta)$ within optimal, where $\delta$ is the maximum number of users that are blocked from receiving due to interference from a given transmission. Throughout the paper, we discuss approaches to increase the speed of the proposed algorithm while having only minimal effect on the maximum achievable capacity.

Our work is also motivated by the desire to compare and contrast the achievable network capacity under a variety of interference patterns, including those for new and emerging wireless technologies. Traditional wireless communications have been modeled using omni-directional antennas [1], with numerous works looking at finding feasible routes and schedules for omni-directional interference patterns [5]. More recently, networks using directional antennas have been studied [6], where energy can be focused towards or received from specific users. New technologies such as smart-antennas with adaptive digital beamforming allow a user to selectively communicate simultaneously with multiple other users by forming either multiple transmit or receive beams, while steering nulls in the direction of users such as to not interfere with them [7]. This adaptive multi-beam communication system allows for almost arbitrary interference patterns between users depending on which set of beams and nulls are activated at any given moment in time. An algorithm for rapidly determining the achievable network capacity under arbitrary interference constraints is needed to allow the investigation of all types of wireless networks, as well as to allow a common mechanism to examine the effect of

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different wireless technologies and their respective interference patterns on network capacity.

The rest of this paper is organized as follows. In Section II, we review of related work in this area. In Section III, we discuss the model and problem description. In Section IV, we present our algorithm for rapidly finding the achievable capacity of a wireless network. In Section V, we present simulation results for our algorithm, where we compare the achievable network capacity for different wireless technologies and interference patterns.

## II. Related Work

A large number of papers have studied the asymptotic behavior of networks. In particular, these works consider network capacity as the number of users goes to infinity. Typically transmission ranges and node placements are configurable, and finding feasible transmission schemes to achieve any capacity bound is usually not addressed. For omni-directional networks without mobility, [1] show that for $n$ users, total network capacity scales at best $O(\sqrt{n})$. In other words, as the number of users in the network grows to infinity, the end-to-end capacity available for any particular flow between users goes to zero. The analysis from [1] has been extended to different types of networks and interference patterns, including mobile networks [8], directional networks [9, 10], and MIMO relay networks [11]. While asymptotic results are useful at understanding fundamental network limits, they often do not say anything about capacities of finite node networks, nor how to actually achieve those capacity bounds.

Numerous papers have been written on the topic of cross-layer optimization that attempt to maximize some network metric (typically capacity) by finding a feasible solution consisting of a set of interference-free routes and schedules for the set of connections. Omni-directional networks are examined in [12–15], with each of these papers providing solutions that are within a guaranteed bound of optimal. Heuristic algorithms for routing and scheduling in directional networks are proposed in [16–18], but none provide any guaranteed bounds. For general interference constraints, [2, 4, 19] provide optimal solutions, but their proposed algorithms are computationally intractable and do not run in any guaranteed amount of time.

More recently, [3] finds an upper bound, as well as an achievable lower bound, on the network capacity by maximizing the multi-commodity flow over the sparsest "wireless" cut. While they demonstrate via simulation that their approach finds good solutions, the proposed algorithm for finding a set of feasible flows has no guarantees with respect to optimal and does not run quickly since the end-criteria is to search the entire solution space, which is exponential in size.

## III. Model and Problem Description

In this paper, we study the problem of quickly determining the maximum achievable network capacity of a multi-hop wireless network subject to general interference constraints. Our goal is to provide a feasible route and schedule for all connections that maximizes the minimum fraction satisfied of each demand[1]. We consider both unicast and multicast flows. For the case of a network with both unicast and multicast flows, polynomial-time algorithms based on linear programming (LP) are developed that achieve a maximum concurrent flow that is $O(\delta)$ from the optimal solution, where $\delta$ is the maximum number of users that are interfered with during any particular transmission. While an LP is polynomial-timed, it does not necessarily solve in an acceptable amount of time. For a network with only unicast flows, we leverage the large body of work developed to rapidly determine the maximum concurrent flow in wired (i.e., interference-free) networks [20] to develop rapid algorithms for calculating network capacity while maintaining strict bounds with respect to the optimal solution. For the multicast case, we propose fast algorithms that perform well in practice.

The network model is as follows. We are given a graph $G$ with a set of wireless nodes $V$, edges $E$, and link capacities $C$. The number of nodes in the network is $n = |V|$, and the number of edges is $m = |E|$. The transmission powers (and correspondingly, transmission distances) of each of the users are known. Additionally, we assume that all values are real and rational.

For mobile networks, we consider examining a stationary snapshot. Hence, we assume that the wireless nodes are static, and the set of edges $E$ is fixed. The number of neighbors node $v$ has is called its *degree*, and the maximum degree in graph $G$ is labeled as $\Delta(G)$. There exists a set of unicast demands $D^u$ and multicast demands $D^m$. For a unicast flow, a connection is formed between two users $i$ and $j$. For a multicast flow, a connection is formed from the source $s$ to all of the multicast members subscribing that particular multicast flow; let $\mathbf{S}_s$ represent the set of users subscribing to a multicast flow originating from $s$. For a unicast demand, $d^u(i,j)$ units of flow must be sent from node $i$ to node $j$. For a multicast demand, $d^m(s,\mathbf{S}_s)$ units of flow are sent from source node $s$ to all multicast members $\mathbf{S}_s$.

The binary interference model is used, which is defined as follows: for any pair of links, $\{i,j\}$ and $\{k,l\}$, either both links can be active simultaneously, or at most one link can be active [21]. Binary interference is used for the $K$-hop interference model [22], and the protocol interference model [1]. In $K$-hop interference, if link $\{k,l\}$ is within $K$ hops of link $\{i,j\}$, the two links will interfere. The 2-hop interference is often used to represent the IEEE 802.11 (Wi-Fi) model [22]. In the protocol model, link $\{i,j\}$ can be active only if $i$ is within range of $j$, and no other nodes that are within range of $j$ are transmitting. For the binary interference model, an interference matrix $\mathcal{I}$ can be defined where $I_{ij}^{kl} \in \mathcal{I}$ is 1 if links $\{i,j\}$ and $\{k,l\}$ can be activated simultaneously (do not interfere with each other), and 0 otherwise.

We assume that the network uses a synchronous time slotted system, with equal length time slots, where the set of time slots is $\mathcal{T}$, and $T = |\mathcal{T}|$. If link $\{i,j\}$ is active during time slot $t$, then $\lambda_{ij}^t = 1$, and is 0 otherwise. The activation time for link

[1] Also known as the Maximum Concurrent Flow Problem [20].

$\{i,j\}$ is $\alpha_{ij} = \frac{1}{T}\sum_{t\in\mathcal{T}} \lambda_{ij}^t$.

In our analysis, we use a *conflict graph* [4], which is used to represent the interference in a network using a binary interference model. We construct a conflict graph $G^c$ as follows: a node is added for each edge in the transmission graph $G$, and an edge is added between two nodes in $G^c$ if the edges associated with those nodes interfere with one another. Any independent set[2] of $G^c$ are a set of edges in the transmission graph $G$ that can be activated simultaneously without interference. The maximum degree of the conflict graph, $\Delta(G^c)$, is the maximum number of links that cannot be activated due to interference from some particular transmission. We define $\delta = \Delta(G^c)$.

## IV. Algorithm for Determining Network Capacity

In this section, we present our algorithm for determining the maximum achievable network capacity. Our approach focuses on first determining the ratio of link usage between links in some maximum concurrent flow, and then assigning time slots proportionally to support that flow. Hence, our algorithm has two main steps: first, we find the maximum concurrent flow in the network assuming there exists no interference (i.e., assume it is a wired network), and second, we find an interference-free schedule for the flow found in step one, with the achieved flow being $O(\delta)$ from the optimal achievable maximum flow. The motivation for first solving the maximum concurrent flow problem on the network without any interference is that that solution will give insight into which links are more heavily utilized in supporting a maximum flow, which will then allow us to assign those links proportionally more time slots.

The maximum concurrent flow for networks without interference for both unicast and multicast flows is well studied; hence, the focus of this paper is presenting an algorithm for quickly determining an interference-free schedule for the maximum concurrent flow in wireless networks subject to interference constraints. In Section IV-A, we assume a mechanism exists to find the maximum concurrent flow in a network without interference constraints, and we present the algorithm for determining an interference-free schedule. In Section IV-B, we discuss various methods of solving for the maximum concurrent flow for networks without interference for both unicast and multicast flows.

### A. Finding an Interference-Free Schedule

In this section, we assume that the maximum concurrent flow for the network without interference constraints has already been found, and the objective is to find an interference-free schedule that maximizes individual link rates while accounting for link interference. In particular, we assume that a flow that supports fraction $F$ of every demand has been already found for a network without any interference constraints, and that we are provided the utilization of any particular link for this solution. We define link utilization as follows: if the provided solution has $f_{ij}$ flow allocated on link $\{i,j\}$, then the link utilization of $\{i,j\}$ is $u_{ij} = \frac{f_{ij}}{c_{ij}}$, where $c_{ij}$ is the capacity of link $\{i,j\}$. We note that the provided solution for maximum concurrent flow $F$ of the network *without* interference constraints is an upper bound on the maximum flow of a network *with* interference.

Next, we define a cross-link utilization ratio between two links $\{i,j\}$ and $\{k,l\}$: $r_{kl}^{ij} = \frac{u_{ij}}{u_{kl}}$. The key to our scheduling approach is to find a time slot activation such that the achieved flow on any link preserves this cross-link utilization ratio. If a final schedule has $T$ time slots and link $\{i,j\}$ uses $t_{ij}$ time slots, then the supportable wireless flow on $\{i,j\}$ is $\alpha_{ij} = \frac{1}{T}t_{ij}$. Hence, our goal is find an interference-free schedule such that $r_{kl}^{ij} = \frac{u_{ij}}{u_{kl}} = \frac{\alpha_{ij}}{\alpha_{kl}}$, $\forall\{i,j\},\{k,l\} \in E$. By guaranteeing that $\alpha_{ij}$ is within $O(\delta)$ of $u_{ij}$, $\forall\{i,j\}\in E$, we can guarantee that the achieved maximum concurrent flow in the wireless network with interference is bounded with respect to the upper bound.

A high-level outline of the algorithm is first presented. Afterwards, each step is then discussed in more detail, as well as potential for speed improvements.

1) Convert all link utilizations $u_{ij}$ to integer values $z_{ij}$ such that the cross-link utilization factor $r_{kl}^{ij}$ is preserved; i.e., $r_{kl}^{ij} = \frac{z_{ij}}{z_{kl}} = \frac{u_{ij}}{u_{kl}}$.
2) Create a conflict graph of the original graph $G$ using the given interference constraints $\mathcal{I}$ as follows. Instead of having link $\{i,j\}$ from $G$ represent one node $v_{ij}$ in the conflict graph, $\{i,j\}$ will be represented by $z_{ij}$ nodes that form a clique[3]; we label this conflict graph $G^Q$. If links $\{i,j\}$ and $\{k,l\}$ cannot be active simultaneously in the original network $G$ (i.e., they interfere with one another), then in $G^Q$, each node from the clique $\{i,j\}$ will have a connection to each node in clique $\{k,l\}$.
3) We next find a minimum graph coloring of $G^Q$, where each color represents a time slot of the final schedule. Since no two nodes of a clique can share the same color, a clique of size $z$ will require exactly $z$ colors. Hence, a final graph coloring preserves the cross-link utilization factor $r_{kl}^{ij} = \frac{z_{ij}}{z_{kl}}$. Since the graph coloring problem is strongly NP-Complete [23], we use the Welsh-Powell algorithm for graph coloring that guarantees a solution that is within $\Delta(G^Q)$ of optimal [24], where $\Delta(G^Q)$ is the maximum degree of the conflict graph $G^Q$.
4) If the total number of time slots to color $G^Q$ is $T$ and link $\{i,j\}$ uses $z_{ij}$ time slots, then link $\{i,j\}$ has a new utilization factor $\alpha_{ij} = \frac{z_{ij}}{T}$, where $\alpha_{ij}$ is the wireless activation ratio. Recall that the utilization factor $u_{ij}$ is the ratio of the flow allocated to link $\{i,j\}$ and the capacity of that link. The wireless activation ratio is the percentage of time that a link can be active. If $\alpha_{ij} \geq u_{ij}$, then the full flow on link $\{i,j\}$ can be supported, and if $\alpha_{ij} < u_{ij}$, then only the fraction $\rho_{ij} = \frac{\alpha_{ij}}{u_{ij}}$ can be supported. Define $\rho_{min} = \min_{\forall\{i,j\}\in E} \rho_{ij}$. We can then scale the maximum concurrent flow for the network without interference by $\rho_{min}$ to find an achievable maximum flow in a network with interference. We will demonstrate that $\rho_{min} \geq \delta^{-1}$,

[2] An independent set is a set of nodes where no two nodes are the end points of the same edge.

[3] A clique are a set of nodes that are all connected to one another.

where $\delta$ is the maximum number of links that cannot be active due to interference with a particular transmission.

We now discuss each step of the algorithm in more detail. In step 1, all link utilizations $u_{ij}$, $\forall\{i,j\} \in E$ are converted to integer values $z_{ij}$. To do so, we find some integer value $R$ such that $R \cdot u_{ij} \in \mathbb{Z}$, $\forall\{i,j\} \in E$, where $\mathbb{Z}$ is the set of integers. This requires all values $u_{ij}$ to be rational. We demonstrate this to be the case in Lemma 1.

**Lemma 1.** *There exists some integer value $R$ such that $R \cdot u_{ij} \in \mathbb{Z}$, $\forall\{i,j\} \in E$, where $R$ is polynomial bounded in size by the size of the input variables.*

*Proof:* As assumed by the network model, all inputs to our problem are rational, including all link capacities $c_{ij}$, $\forall\{i,j\} \in E$. As will be shown in Section IV-B, the maximum concurrent flow for networks without interference can solved using a linear program (LP). The output of the LP are a set of flow allocations on each edge: $f_{ij}$, $\forall\{i,j\} \in E$. In an optimal solution given by an LP, the size of any output variable (i.e., the number of bits necessary to represent that variable) is polynomially bounded by the size of the inputs [25]. Since all of the inputs to our problem are rational (hence requiring a finite and bounded number of bits to represent), all output variables $f_{ij}$, $\forall\{i,j\} \in E$ are also rational and polynomial bounded by the size of the inputs. Since link capacities are rational, the link utilization $u_{ij} = \frac{f_{ij}}{c_{ij}}$ is also rational. Therefore, there exists some value $R$ that is polynomially bounded by the size of inputs such that $R \cdot u_{ij} \in \mathbb{Z}$, $\forall\{i,j\} \in E$. ∎

We note that demonstrating the existence of an $R$ that is polynomial-bounded in size with respect to the inputs is important from an analytic perspective, but not necessarily from a practical perspective. The value $R$ is a scaling factor used to determine the size of cliques for the conflict graph. Since the runtime of any graph coloring algorithm is dependent on the number of nodes in the graph, having a large value of $R$ can result in a large number of nodes which results in slower algorithm performance. Smaller values of $R$ can be used (dropping any fractional values) and still produce high-fidelity results while experiencing significantly faster algorithm runtimes.

Next we discuss steps 2 and 3 of the algorithm. A typical conflict graph construction represents any individual link $\{i,j\}$ in $G$ as a single node $v_{ij}$; we call this conflict graph construction $G^1$. In our solution approach, we construct an alternative conflict graph where each link $\{i,j\}$ in $G$ is represented by $z_{ij}$ nodes that form a clique. Since no two nodes of a clique can share the same color, a clique of size $z$ will require exactly $z$ colors. Recall that that $z_{ij} = R \cdot u_{ij}$. Hence, a final graph coloring preserves the cross-link utilization factor $r_{kl}^{ij} = \frac{R \cdot u_{ij}}{R \cdot u_{kl}} = \frac{z_{ij}}{z_{kl}}$. If $R$ is reduced in size and the fractional value is discarded, the cross-link utilization factor $r_{kl}^{ij}$ is roughly preserved. A trade-off in exact precision versus runtime may be desired in practice.

In step 4, we scale the initial maximum concurrent flow for the network without any interference constraints such that it can be supported by the interference-free schedule that was found in step 3. If the total schedule has $T$ time slots, and link $\{i,j\}$ has $z_{ij}$ time slots assigned to it, $\{i,j\}$ will have an activation ratio of $\alpha_{ij} = \frac{z_{ij}}{T}$. If $\alpha_{ij} \geq u_{ij}$, then the full flow on link $\{i,j\}$ can be supported, and if $\alpha_{ij} < u_{ij}$, then only the fraction $\rho_{ij} = \frac{\alpha_{ij}}{u_{ij}}$ can be supported. We define $\rho_{min} = \min_{\forall\{i,j\}\in E} \rho_{ij}$. We can scale the initial solution to the maximum concurrent flow without interference by $\rho_{min}$ to find an achievable maximum concurrent flow in the wireless network with interference constraints.

We now demonstrate that $\rho_{min} \geq \delta^{-1}$, and hence our algorithm always produces a solution that is $O(\delta)$ of optimal. We formally define $\delta$ as the maximum degree of $G^1$. Recall that in this conflict graph, each link $\{i,j\}$ of $G$ is represented by one node $v_{ij}$ in $G^1$, and two nodes in $G^1$ have a connection if and only if they cannot be activated simultaneously. Hence, the maximum degree of the conflict graph $G^1$ is the maximum number of links that cannot be activated due to interference to some particular transmission, where we define the maximum degree of $G^1$ as $\delta = \Delta(G^1)$.

**Theorem 1.** *Our algorithm finds a feasible maximum concurrent flow for a network with interference constraints that is always within $\delta$ of the maximum concurrent flow when interference constraints are not considered.*

*Proof:* In particular, we will demonstrate that $\frac{\alpha_{ij}}{u_{ij}} \geq \delta^{-1}$, $\forall\{i,j\} \in E$, where $\alpha_{ij} = \frac{z_{ij}}{T}$. We define the following two values: $z_{max} = \max_{\forall\{i,j\}\in E} z_{ij}$, and $u_{max} = \max_{\forall\{i,j\}\in E} u_{ij}$.

By using the Welsh-Powell graph coloring algorithm, conflict graph $G^1$ can be colored using $\Delta(G^1)$ colors [24], where $\Delta(G^1)$ is the maximum degree of $G^1$. Recall that $\delta = \Delta(G^1)$. In the clique version of the conflict graph, $G^Q$, any particular clique of size $z$ will be colored using $z$ colors. Hence, $G^Q$ can be colored with at most $\delta z_{max}$ colors.

To compute $z_{ij}$ for any particular link, we multiplied each link utilization ratio $u_{ij}$ by some value $R$: $z_{ij} = R \cdot u_{ij}$. This implies $z_{max} = R \cdot u_{max}$. Recall that the link utilization ratios $u_{ij}$ were the total percentage of link capacity that was used to support the maximum concurrent flow in the network without interference. Maximum concurrent flow is achieved when the multi-commodity minimum-cut is saturated, and the minimum-cut is saturated when all of its respective links are allocated at capacity [26]. Hence, $u_{max} = 1$, and $z_{max} = R \cdot u_{max} = R \cdot 1 = R$.

Using $T \leq \delta z_{max}$, the link activation ratio has the following bound: $\alpha_{ij} = \frac{z_{ij}}{T} \geq \frac{z_{ij}}{\delta z_{max}}$, $\forall\{i,j\} \in E$. We rewrite the link utilization factor as $u_{ij} = \frac{z_{ij}}{R}$. Using $R = z_{max}$, we complete the proof:

$$\frac{\alpha_{ij}}{u_{ij}} \geq \frac{\frac{z_{ij}}{\delta z_{max}}}{\frac{z_{ij}}{R}} = \frac{R}{\delta z_{max}} = \delta^{-1},\ \forall\{i,j\} \in E$$

∎

### B. Finding the Maximum Concurrent Flow for a Network without Interference Constraints

In the previous section, we demonstrated that when given a maximum concurrent flow for a network without interference constraints, our scheduling algorithm finds an interference-free schedule that supports $O(\delta)$ of the original maximum flow. The scheduling algorithm is agnostic to whether the flows are unicast and multicast, or how those flows were calculated. In this section we present different approaches for calculating the maximum concurrent flow without interference constraints.

The maximum concurrent flow problem aims to maximize the minimum fraction of each connection that can be supported in a capacitated network. There is a large body of literature that we can leverage for computing the maximum concurrent flow for both the unicast and multicast case. A linear program to optimally solve for the maximum concurrent flow problem has been previously provided in [27] for the unicast case and in [28] for the multicast case. In both of those papers, the linear programming formulations find optimal solutions in polynomial time.

While linear programs provide optimal solutions, they are not necessarily efficient to solve in practice. The authors of [28] benchmarked the performance of their linear programming formulation for the multicast maximum concurrent flow and found that runtime can easily take hours for moderately sized networks.

In [20], a survey of approximation algorithms for the unicast maximum concurrent flow problem are presented, as well as a new algorithm that performs even faster. To calculate a $(1+\epsilon)$ approximation of the optimal unicast maximum concurrent flow, [20] develops an algorithm that runs in $O(\epsilon^{-2}(k+m)m)$ time, where $k$ is the number of unicast connections and $m$ is the number of edges in the network.

For the multicast case, [28] provide an algorithm that achieves the optimal solution. Their approach does not have guaranteed polynomial runtime, but in practice achieves the optimal solution rapidly. Their simulation results show the multicast maximum concurrent flow can solved in under one second for networks approaching 1000 nodes in size. We note that their approach can be used for the unicast case as well: a multicast session from source $s$ to a single destination $d$ is identical to a unicast flow between $s$ and $d$. The algorithm in [28] provides a viable approach to rapidly calculate the optimal maximum concurrent flow for both the unicast and multicast case.

## V. Algorithm and Network Performance Evaluation

In this section we evaluate the performance of the developed algorithm and use our algorithm to compare network capacities for different wireless technologies that operate using different interference constraints. In Section V-A, we evaluate the performance and runtime of the algorithm. In Section V-B, we compare the maximum achievable throughput for networks using different wireless technologies. In particular, we compare networks utilizing omni-directional antennas operating at 1 GHz ISM band, and single-beam directional, and multi-beam adaptive directional antennas utilizing different beam widths operating at the higher frequency 15 GHz ISM band. For directional networks, different beam widths will produce different gain and interference patterns. To the best of our knowledge, this paper is the first to evaluate achievable network capacity for the various wireless technologies that takes into account appropriate interference parameters.

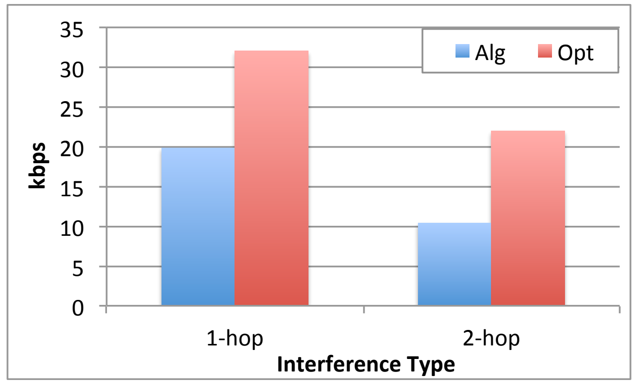



Fig. 1: Average throughput per connection: 100 node network with all-to-all unicast demands

TABLE I: Average runtime for 100 node networks

| Interference Type | Average Runtime (sec) |
|---|---|
| 1-Hop | 0.21 |
| 2-Hop | 0.94 |

### A. Algorithm Evaluation

To evaluate the algorithm, we compare its performance to that of the optimal solution. We consider the set of all-to-all unicast demands in a network of 100 nodes. For this particular set of tests, fifty random graphs are generated, and all link capacities are set to 1 Mbps. We consider two interference models: one-hop and two-hop. For one-hop interference, also referred to as the node exclusive model, a node can be either only transmitting or receiving, but not both. For two-hop interference, a node can only be transmitting or receiving if no node within two hops is active; two-hop interference is sometimes used to represent interference for the IEEE 802.11 Wi-Fi model [22]. For the optimal solution, we implement the algorithm developed in [19] that utilizes back-pressure routing and scheduling. As discussed in Section II, [19] develops a solution that can be used to optimally solve for the interference-aware maximum concurrent flow, but it does not run in a polynomial amount of time.

The results of the simulation are shown in Figure 1. Our algorithm achieves on average a maximum concurrent flow that is approximately half of optimal. Algorithm runtimes are shown in Table I. For both one-hop and two-hop interference, average algorithm runtime is below one second. For one-hop interference, runtime was 0.21 seconds, and for two-hop interference, runtime was 0.94 seconds. In contrast, the optimal

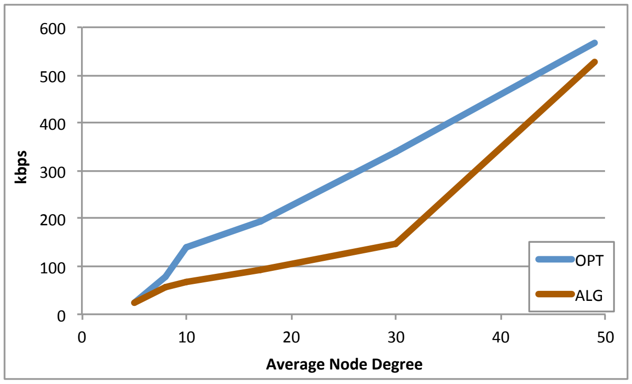



Fig. 2: Rates achieved by algorithm vs. the optimal solution: 50 node network

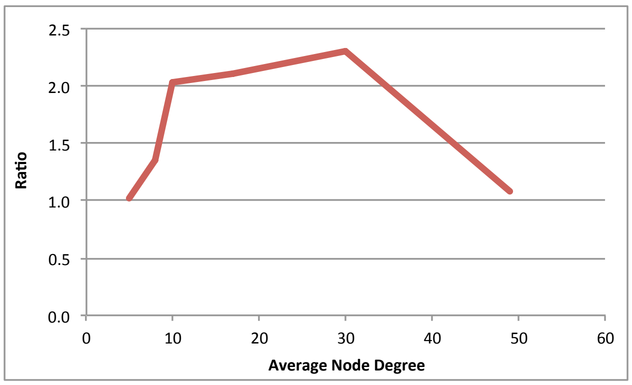



Fig. 3: Ratio of algorithm with the optimal solution: 50 node network

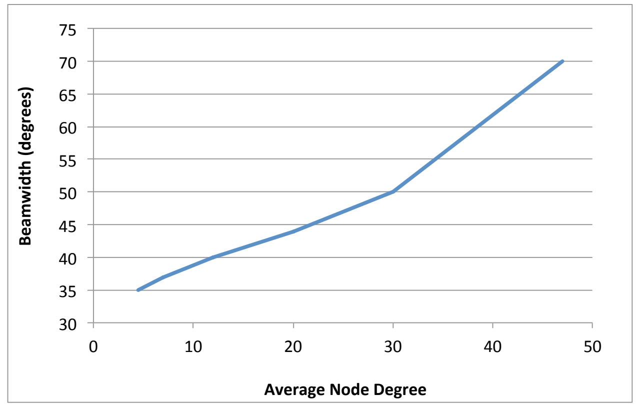



Fig. 5: Break even point for 50 node network: 15 GHz Directional vs. 1 GHz Omni

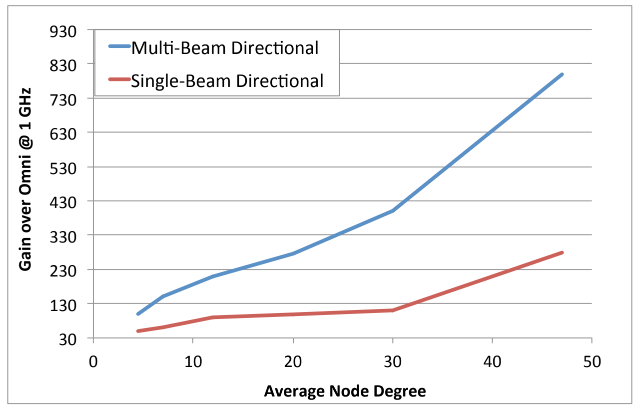



Fig. 6: 15 GHz Directional gain over 1 GHz Omni in a 50 node network

solution took four days to run for each particular instance.

We next consider the algorithms performance on networks that can support different rates on the various links. The achievable rate of a wireless channel between two users has a large number of factors that must be considered. For simplicity, we assume that the possible link rate between two users is a function of the transmitter's power and the distance between the transmitter and receiver. For this set of tests, we consider 50 node networks, and we vary the density of users (i.e., we vary the average node degree). All users transmit at 1 Watt, and free-space path-loss is assumed. We use the one-hop interference model for our tests.

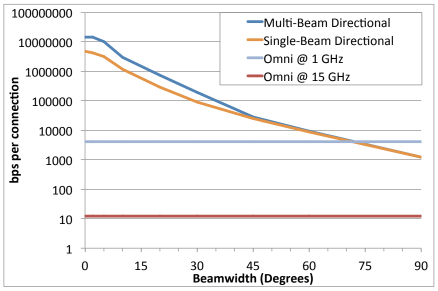
The achieved maximum concurrent flow is plotted in Figure 2, and the ratio of optimal solution to the algorithm's solution is plotted in Figure 3. For dense networks, users are close together, and the achievable rates are very high; when users are farther apart, rates are lower. In dense networks, more interference will be experienced by different users for some given transmission, and the link activation time will decrease; this potentially offsets the higher achievable rates that are possible for users that are within close range of one another. In Figure 2, we can see that when users are clustered together, the average rate that can be achieved is above 500 kbps. But as density decreases, the achievable maximum concurrent flow decreases as well. At the network's sparsest density, where the average node degree is 4, the achievable rate is 23 kbps. So, while dense network may experience higher interference, the benefit from higher achievable link rates seems to outweigh the loss due to interference.

With respect to the algorithm's performance, we see that at the algorithm performs almost as well as optimal in the extreme cases when the network has very high and very low density. For the other cases, the algorithm provides a solution that is at most a factor of 2.4 from optimal.

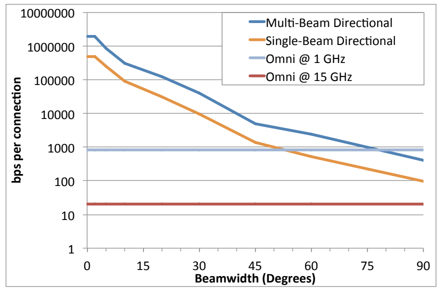
### B. Network Performance Evaluation

We now consider the achievable capacity for networks using different wireless technologies. For omni-directional antennas, we assume zero gain. For directional antennas, gain is a function of the achieved beam width: the narrower the beam width, the higher the gain, thus the higher the link rate between the two users. Additionally, narrower beams means there will be lower interference between two users. The directional networks operate at 15 GHz, and the omni directional networks operate at 1 GHz.

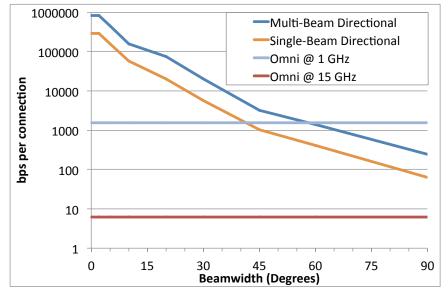
For this set of tests, we consider 50 node networks, and we vary the beam width (which varies the gain), and we vary the density of users. All users transmit at 1 Watt, and free-space



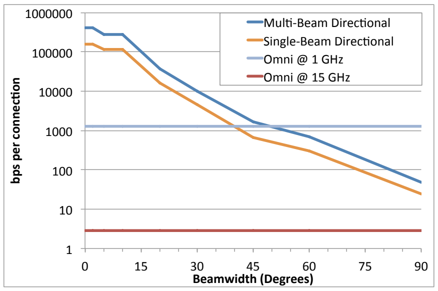

(a) Average Node Degree: 47



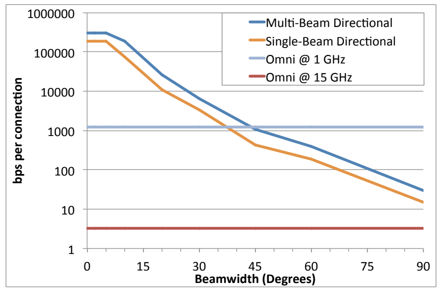

(b) Average Node Degree: 30



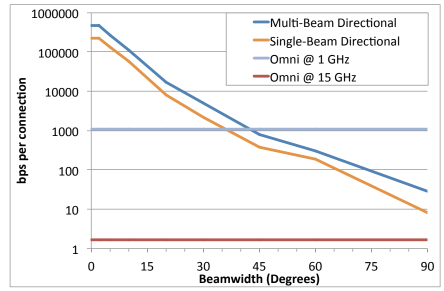

(c) Average Node Degree: 17




(d) Average Node Degree: 10




(e) Average Node Degree: 7




(f) Average Node Degree: 4

Fig. 4: Networks with different node degrees

path-loss is assumed. We assume triangular shaped beam patterns where transmissions have some maximum distance, and we ignore the effect of side lobes. Interference constraints are generated accordingly. We note that different signal processing algorithms can suppress side lobes at a slight cost of beam width. A multi-beam directional system is capable of either transmitting or receiving multiple beams simultaneously.

Results for the different scenarios are plotted in Figure 4. We define a "break-even" point, where the beam width for the directional beam becomes so wide that the same performance can be achieved using omni directional antennas at a lower frequency. The break even point for various network densities is plotted in Figure 5. The ratio improvement in achievable network rate of the directional network over the omni-directional network is plotted in Figure 6.

## VI. Conclusion

This paper considers the problem of rapidly determining the maximum achievable capacity of a multi-hop wireless mesh network subject to interference constraints. Being able to quickly calculate the maximum supported flow in a wireless network has numerous practical applications for network planners and researchers, including quickly visualizing capacity limits of a wireless network, benchmarking the performance of wireless protocols, and rapidly determining the instantaneous capacity of various network topologies or different snapshots of a mobile network. Current approaches for determining network capacity either provide asymptotic results that are not necessarily achievable, are computationally intractable, or are not generalizable to different interference constraints for emerging technologies. In this paper, we present a new algorithm to rapidly determine the maximum concurrent flow for an arbitrary number of unicast and multicast connections subject to general interference constraints, and provide a feasible route and schedule for those flows. The solution provided by our algorithm is within $O(\delta)$ of the optimal maximum flow, where $\delta$ is the maximum number of users that are blocked from receiving due to interference from a given transmission. We then use our algorithm to perform a network capacity analysis comparing different wireless technologies, including omni-directional, directional, and multi-beam directional networks.

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